Full-Wave Analysis of Cross-Aperture Waveguide Couplers

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***Abstract*—This paper presents a full-wave analysis of coupling between rectangular waveguides through a cross-aperture. The rigorous mode-matching method is used to derive the generalized scattering matrix of a waveguide T-junction having a crossed waveguide as the side arm. Two three-port T-junctions are then cascaded together to form a number of cross-aperture couplers. The analysis method can handle both broad-wall and narrow-wall, both parallel and crossed coupling structures. Our numerical results for a variety of couplers are in good agreement with those obtained by Ansoft's HFSS.**

***Index Terms*—Cross-aperture, mode-matching method, waveguide coupler.**

I. Introduction

WAVEGUIDE coupling problem is a fundamental electromagnetic problem [1]–[3], which has found a variety of applications [4], [5], such as directional couplers, filters, power dividers, and combiners, etc. Coupling between waveguides can take place through apertures of various shapes, for example, rectangular-aperture [5], circular-aperture [5], and cross-aperture [6], [7]. The cross-aperture is sometimes preferred to realize flat coupling and high directivity over a wide frequency range. This is evidently seen in designing compact and high performance Moreno coupler [6]. Cross-aperture can also be very useful in slot antennas of circular polarization.

Bethe's small-hole coupling theory [1] is perhaps the earliest solution to the coupling problem through a cross-aperture. However, it can only be applied to small aperture with zero thickness. Even with Cohn's correction [2] and McDonald's improvement [3], the theory is still approximate and can only be useful in designing directional couplers of relatively weak coupling [6]. In 1998, Park and Nam [7] applied the finite-element boundary-integral method to analyze the arbitrary-shaped multiaperture-coupled directional coupler. But, this method is computationally time-consuming and cannot be used for synthesis purpose.

This paper presents a mode-matching analysis of the waveguide coupling problem through a cross-aperture. The mode-matching method is formally exact and computationally very efficient. Unlike the cascading technique used in [5], we use the mode-matching method to derive the generalized scattering matrix of a waveguide T-junction having a crossed waveguide as the side arm. Two three-port T-junctions are then cascaded together to form the coupling structure. This treatment greatly reduces the matrix operations involved in the cascading process [5] and in turn improves the efficiency. This method can readily handle both broad-wall and narrow-wall rectangular waveguide couplers via cross apertures. The two rectangular waveguides can be either parallel or crossed.

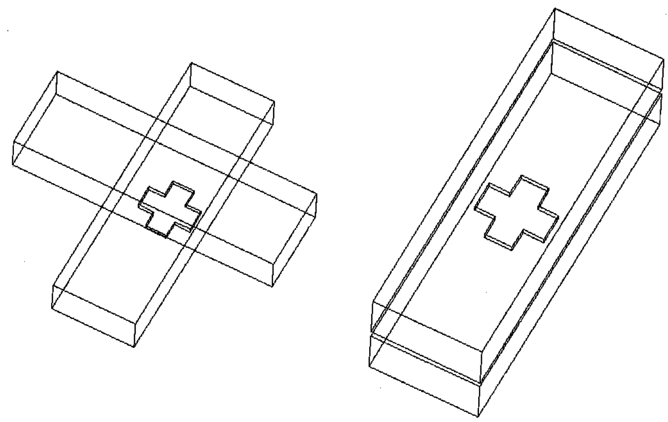

Fig. 1. Rectangular waveguide cross-aperture couplers.

II. Formulation

Fig. 1 shows the geometries of rectangular waveguide couplers involving cross-aperture.

In order to analyze the couplers shown in Fig. 1, we need derive the generalized scattering matrix of a basic building block—a waveguide T-junction having a crossed waveguide as the side arm. Once this is accomplished, two three-port waveguide T-junctions can be cascaded together to form the four-port cross-aperture couplers of different orientations.

Fig. 2 illustrates the waveguide T-junction having a crossed waveguide side arm. The whole T-junction can be divided into four regions: 1, 2, 3, and 4, as shown in Fig. 2. Regions 1 and 2 are semi-infinite rectangular waveguides, where the expansion expressions of the electromagnetic fields can be easily obtained. Region 3 is a semi-infinite crossed waveguide [its cross section is illustrated in Fig. 2(b)], whose modal functions and cutoff wavenumbers are calculated in [8], [9].

The electromagnetic fields in the three semi-infinite waveguides can be expressed as [10]

$$\vec{E}^{(v)} = \frac{1}{j\omega\varepsilon} \nabla \times \nabla \times \vec{A}_e^{(v)} + \nabla \times \vec{A}_h^{(v)} \quad (1a)$$

$$\vec{H}^{(v)} = \frac{1}{-j\omega\mu} \nabla \times \nabla \times \vec{A}_h^{(v)} + \nabla \times \vec{A}_e^{(v)} \quad (1b)$$

Manuscript received November 15, 2001; revised April 22, 2002. The review of this letter was arranged by Associate Editor Dr. Shigeo Kawasaki.

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Publisher Item Identifier 10.1109/LMWC.2002.801128.

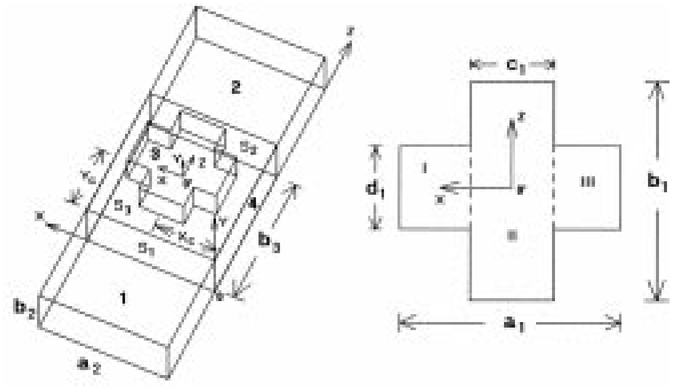

Fig. 2. Geometry of a waveguide T-junction with a crossed waveguide side arm.

where $v = 1, 2, 3$, $\vec{A}_h$, and $\vec{A}_e$ are the vector potentials for TE and TM modes, respectively. For Region 3, we can have the following form for $\vec{A}_h^{(3)}$ and $\vec{A}_e^{(3)}$:

$$\vec{A}_h^{(3)} = \vec{a}_y \sum_{j=1}^{N_h} Q_{hj}^{(3)} T_{hj}^{(3)} \left[A_{3hj}^{+} e^{-\gamma_{3hj} y} + A_{3hj}^{-} e^{+\gamma_{3hj} y} \right] \quad \text{(2a)}$$

$$\vec{A}_e^{(3)} = \vec{a}_y \sum_{j=1}^{N_e} Q_{ej}^{(3)} T_{ej}^{(3)} \left[A_{3ej}^{+} e^{-\gamma_{3ej} y} - A_{3ej}^{-} e^{+\gamma_{3ej} y} \right] \quad \text{(2b)}$$

where A_{3j}^{+}, A_{3j}^{-} are the modal amplitude coefficients of the reflected $(+)$ and incident $(-)$ waves in the side arm; γ_h and γ_e are the propagation constants of the TE and TM modes, respectively; j is the mode index; Q is the normalized factor such that the power carried by each mode is $1W$ for propagating modes, jW for evanescent TE modes, and $-jW$ for evanescent TM modes; $T_{hj}^{(3)}$ and $T_{ej}^{(3)}$ are the TE and TM modal functions for the crossed waveguide, which, according to the coordinates shown in Fig. 2(b), have the form of (3) and (4), shown at the bottom of the page.

It should be mentioned that we choose $K = 2$ and $M = 100$ for the expansion series in (3) and (4) to ensue convergent results for computing the eigenvalue Kc. This is because d_1 is usually small and the condition $M/K \geq b_1/d_1$ can always be satisfied for the cross-aperture couplers considered.

For Region 4, the resonator method [11], [12] is employed to obtain the electromagnetic fields by superimposing three suitably chosen standing-wave solutions.

Once all the field expansions for all the four regions are available, we then match the tangential electric and magnetic fields along the three regional interfaces S_1, S_2, and S_3. The relationship between the modal amplitude coefficients ($A_{(v)}^{+}$ and $A_{(v)}^{-}$, $v = 1, 2, 3$) of the three semi-infinite waveguides can be derived to result in the generalized scattering matrix of the waveguide T-junction.

The waveguide couplers shown in Fig. 1(a)–(b) can be formed by cascading two T-junctions with a crossed waveguide side arm illustrated in Fig. 2(a). For this purpose, the standard generalized scattering matrix technique [13] is employed to obtain the overall scattering matrix of coupling structures.

It can be seen from Fig. 2 that the length b_3 of the resonator can be a variable, as long as $b_3 \geq b_1$. It is noted that $b_3 = b_1$ should be recommended to obtain efficient and convergent results.

III. Numerical Results

Fig. 3 shows the calculated scattering parameters of a coupling structure with a cross-aperture located at the center of the broad-wall of two parallel rectangular waveguides (WR62: $a_2 \times b_2$ = 15.799 × 7.899 mm^2). The dimensions of the cross-aperture are: $a_1 = b_1 = 7.899$ mm, $c_1 = d_1 = 2.0$ mm, $t = 1.0$ mm. In this example, it is noted that the cross-aperture is quite large, where Bethe's small-hole theory is not valid. It can be

$$T_{hj}^{(3)} = \begin{cases} \displaystyle\sum_{k=0}^{K} A_k \cos k_{x\mathrm{I}k} \left(x - X_C - \frac{a_1}{2}\right) \cos \frac{k\pi \left(z - Y_C + \frac{d_1}{2}\right)}{d_1} & \text{Region I} \\ \displaystyle\sum_{m=0}^{M} A_m \begin{bmatrix} \sin k_{x\mathrm{II}m}(x - X_C) \\ \cos k_{x\mathrm{II}m}(x - X_C) \end{bmatrix} \cdot \cos \frac{m\pi \left(z - Y_C + \frac{b_1}{2}\right)}{b_1} & \begin{matrix} \text{Region II, Odd} \\ \text{Region II, Even} \end{matrix} \\ \displaystyle\sum_{k=0}^{K} A_k \begin{pmatrix} -1 \\ 1 \end{pmatrix} \cdot \cos k_{x\mathrm{I}k} \left(x - X_C + \frac{a_1}{2}\right) \cos \frac{k\pi \left(z - Y_C + \frac{d_1}{2}\right)}{d_1} & \begin{matrix} \text{Region III, Odd} \\ \text{Region III, Even} \end{matrix} \end{cases} \quad (3)$$

$$T_{ej}^{(3)} = \begin{cases} \displaystyle\sum_{k=1}^{K} B_k \sin k_{x\mathrm{I}k} \left(x - X_C - \frac{a_1}{2}\right) \sin \frac{k\pi \left(z - Y_C + \frac{d_1}{2}\right)}{d_1} & \text{Region I} \\ \displaystyle\sum_{m=1}^{M} B_m \begin{bmatrix} \cos k_{x\mathrm{II}m}(x - X_C) \\ \sin k_{x\mathrm{II}m}(x - X_C) \end{bmatrix} \cdot \sin \frac{m\pi \left(z - Y_C + \frac{b_1}{2}\right)}{b_1} & \begin{matrix} \text{Region II, Odd} \\ \text{Region II, Even} \end{matrix} \\ \displaystyle\sum_{k=1}^{K} B_k \begin{pmatrix} -1 \\ 1 \end{pmatrix} \cdot \sin k_{x\mathrm{I}k} \left(x - X_C + \frac{a_1}{2}\right) \sin \frac{k\pi \left(z - Y_C + \frac{d_1}{2}\right)}{d_1} & \begin{matrix} \text{Region III, Odd} \\ \text{Region III, Even} \end{matrix} \end{cases} \quad (4)$$

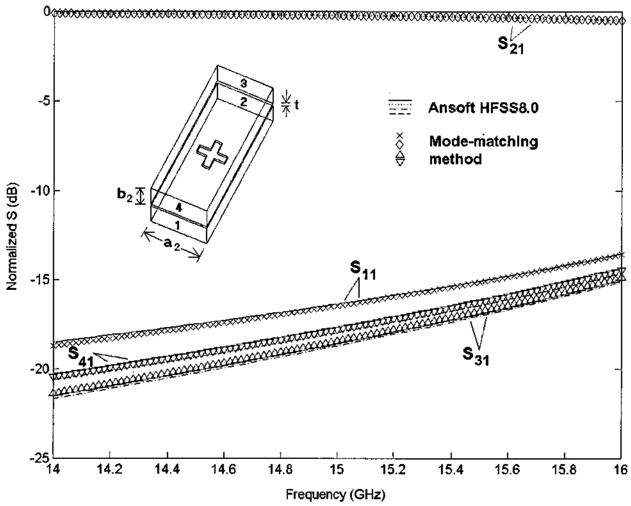

Fig. 3. Scattering parameters of a broad-wall cross-aperture coupler.

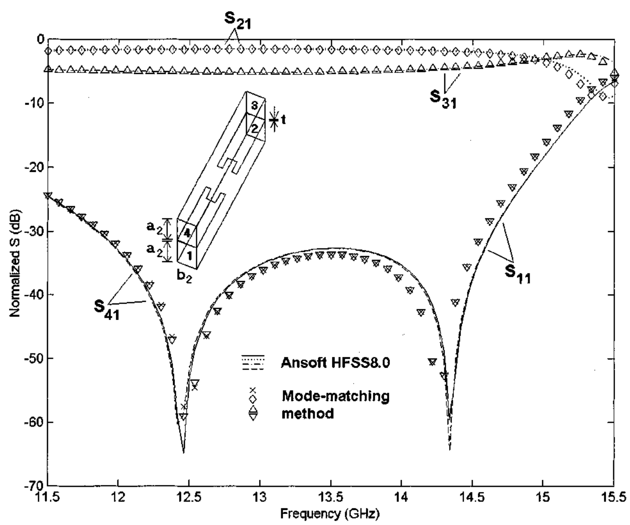

Fig. 4. Scattering parameters of a narrow-wall cross-aperture directional coupler.

seen from Fig. 3 that our mode-matching results are in excellent agreement with those computed by HFSS. In our calculation, 30 modes are considered in the crossed waveguide and 50 modes are retained in the main arm.

Narrow-wall couplers are frequently used in high-power applications [5]. Fig. 4 gives an example of such coupler utilizing cross-aperture. A flat coupling of -5.0 dB has been achieved (S_{31} in Fig. 4) over a wide band with isolation $>$33 dB. The coupler is designed at Ku band and standard WR62 waveguides are utilized. The dimensions for the cross-aperture in Fig. 4 are $a_1 = 7.899$ mm, $b_1 = 19.0$ mm, $c_1 = 3.8$ mm, $d_1 = 9.8$ mm, $t = 0.02$ mm.

A crossed-waveguide broad-wall-to-broad-wall directional coupler is widely used due to its compact structure and the cross-aperture is usually employed to offer flat coupling and high isolation. Fig. 5 gives an example of such coupler designed at Ku band utilizing WR62 waveguides. By employing only

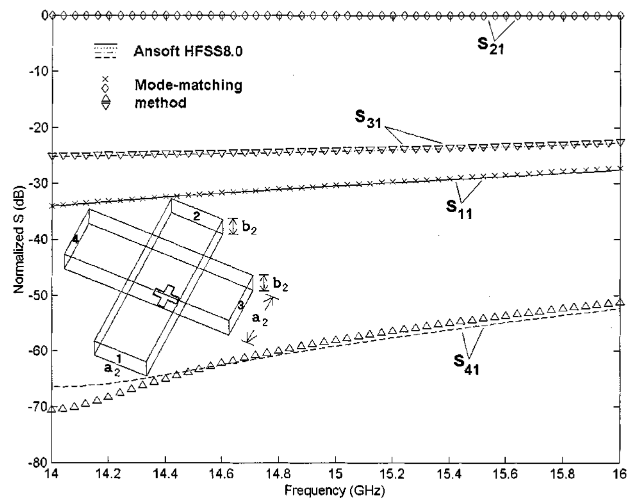

Fig. 5. Scattering parameters of a cross-aperture crossed-waveguide broad-wall directional coupler.

one cross-aperture, we have successfully achieved -24 dB (S_{31} in the Fig. 5) coupling with directivity $>$35 dB. By narrowing down the width of the aperture, a higher directivity could be obtained. The dimensions for the cross-aperture are $a_1 = b_1 = 7.0$ mm, $c_1 = d_1 = 2.0$ mm, $t = 0.5$ mm, $X_c = Y_c = 3.949\,75$ mm.

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